GAME THEORY
ECON 280
FALL 2017
Office Hours: Mon. 10:15 – noon & Thur. 10:45 - noon

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OVERVIEW

The aim of *game theory* is to investigate and understand the manner in which rational people *should* interact when they have complimentary or conflicting interests. So far, game theory only provides answers in simple situations, but these answers have already led to a fundamental restructuring of the way economists think about the world. And, the time is not too distant in the future when the same will be true for all the social (and behavioral) sciences.

In this class, we will examine the theory of games in detail. In the first half of the class we study the basics of the theory including what a game is and how simple games are solved. In the second half of the class we examine more complicated games and applications. Along the way, we may also study some experiments that have been conducted to test the ability of game theory to predict the outcome of interactions between real (and perhaps not rational) people.

TEXT

No text book is required for the course but if you would like a text to read I've made one available on reserve at the main library that follows the course pretty well. It is: *Game Theory: An introduction* by Steven Tadelis.

GRADING

Your performance in the class will depend on three things: Homework Assignments, Midterms, and a Final Exam. Midterm exams are not cumulative, but the final exam is. This means that you will need to study all the topics we cover in the class for the final exam. The in-class exam dates are written on the class schedule (see the back of this sheet) and are not negotiable. There will also be no make-up exams.

However, people may get sick and miss an exam. With this in mind, all four exams (the 3 midterms and the final) will be comparable in length (they will all be designed to be completed in 75 minutes) and only your best 3 of the 4 exams will count. If you cannot make it to one of the 4 exams, for any reason, that will be the exam you drop. Otherwise, if by the end of the course you are happy with your exam grade based on the 3 midterms, you do not have to take the final – after all economics is about choice.

Everything you do for this course will be graded on a 100-point scale. Your final grade for the course will be determined according to the percentages listed below and your performance relative to the performance of the class as a whole.

Your numerical class grade = (0.1 ×Homework)+(0.3 ×Exam 1 Grade)+(0.3 ×Exam 2 Grade)+(0.3 ×Exam 3 Grade)

After returning each exam, I will provide you with a grade distribution for the class. I will assign individual grades relative to the overall performance of the class. In general, I look for natural break points in the range of grades, but the average on any given component of the class will usually be close to a B.

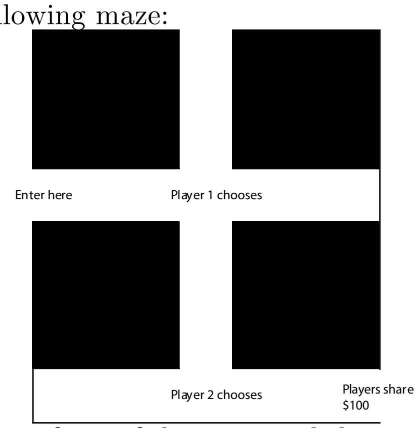
On the back of this sheet you will find a schedule of topics for the course. Behind that I have appended all the problem sets for the class. The due dates for each of the problem sets are included on the class schedule.

Schedule of Topics			
Week	**Dates**	**Homework due**	**Topic**
1	9/12		Introduction to the theory of games
1	9/14		Introduction to the theory of games
2	9/19	#1	Nash Equilibrium for two-person games
2	9/21		Nash Equilibrium for two-person games
3	9/26	#2	Mixed strategies and correlated equilibrium
3	9/28		Mixed strategies and correlated equilibrium
4	10/3	#3	N – person games
4	10/5		N – person games
5	10/10	#4	**First Midterm Exam (in class)**
5	10/12		Market structure
6	10/17		Markets continued
6	10/19		Credibility and subgame perfection
7	10/24		**NO CLASS (Midterm break)**
7	10/26	#5	Subgame perfection continued
8	10/31		Repeated games
8	11/2	#6	**Second Midterm Exam (in class)**
9	11/7		Repeated games continued
9	11/9	#7	Evolutionary game theory
10	11/14		Evolutionary games continued
10	11/16		Signaling and sequential equilibrium
11	11/21	#8	**Third Midterm Exam (in class)**
11	11/23		**NO CLASS (Thanksgiving)**
12	11/28		Signaling continued
12	11/30		Principal – agent games
13	12/5	#9	Auctions
13	12/7		Auctions
14	12/15	#10	Finals week

------------------ EC280 PROBLEM SET 1 --

Please answer all the following questions and try to write legibly.

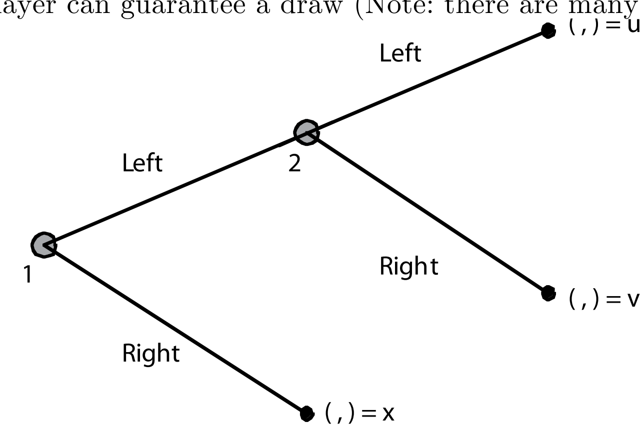
1) Consider the following maze:



Draw the extensive form of the maze and then solve it. Note that player 1 chooses first and then player 2 chooses.

2) Transform the game from problem one from the extensive form into the normal form. Next, find a weakly dominant strategy for each player. Finally, show that the game is weakly dominance solvable by finding its solution.

3) Consider the following extensive form game, with unspecified payoffs. Find payoff vectors U, V, and X such that (a) player 1 can guarantee a win; (b) player 2 can guarantee a win; and (c) either player can guarantee a draw (Note: there are many correct answers).



4) The landlord and the eviction notice:
A landlord has three tenants, A, B, and C, in a rent-controlled apartment building in NYC. A new law says that the landlord has the right to evict one tenant per building. The landlord calculates that the value of a vacant apartment is $15,000, both to the tenant and to her. She sends the following letter to each of the tenants: "Tomorrow I will visit you building. I will offer A $1000 if he agrees to vacate his apartment voluntarily; otherwise, I will evict him. If A agrees to vacate, I will offer $1000 to B, and if she refuses, I will evict her. If she accepts, I will evict C."

(a) Write a game tree for this situation, and find the dominant strategy equilibrium.
(b) What if there were 10 tenants instead of three?
(c) What if the landlord offers $10 instead of $1000.

5) The second-price auction:
A single object is to be sold at auction. There are n>1 bidders, each submitting a single bid,

in secret, to the seller. The value of the object to bidder i is v_i. The winner of the object is the highest bidder, but the winner only pays the second highest bid.

(a) Show that "truth-telling" (i.e., each player bids v_i) is a weakly dominant strategy for each player. Assume, where convenient, there are no ties.
(b) Does the analysis depend on whether the other bidders tell the truth too?
(c) Can you speculate about why real bidders might not behave this way?

6) What is the dominant strategy in the English or ascending price auction (this is the standard auction mechanism you typically think of)? Assume that bidder i bids b_i and has a private value of v_i.

7) Recall that one can find equilibria by the method of iterative deletion of strictly dominated strategies. Consider the following game: In a football game, the offense has two strategies, Run or Pas. The defense has three strategies: Counter Run, Counter Pass or Blitz. The expected payoff (in yards gained) to each combination appears below. Use elimination of dominated strategies to find an equilibrium in this game (list the order in which you eliminate strategies).

		Defense		
		Counter Run	Counter Pass	Blitz
Offense	Run	3, -3	7, -7	15, -15
	Pass	9, -9	8, -8	10, -10

------------------ EC280 PROBLEM SET 2 --

Please answer all the following questions and try to write legibly.

1) Consider a zero-sum game named "Battle of the Networks" modeled on network competition for viewers. There are two networks and they can either show a sitcom or a sports program. They choose programming simultaneously. To think of the payoffs, the networks only care about how much more viewership they receive than the other network. The following normal form game represents the game:

	Sitcom	Sports
Sitcom	10%, -10%	4%, -4%
Sports	0, 0	-10%, 10%

When both networks show a sitcom, network 1 receives 55% of the viewers and network two 45% so network 1 has a 10% advantage. The other payoffs are calculated similarly.

(a) What is the Nash equilibrium of the game?

(b) Suppose that if both networks show a sitcom that network1 receives 66% of the viewers and there are no other changes. Draw the new (zero-sum) normal form. Does the equilibrium change?

2) In a very simple version of 1-card stud poker, the deck is only aces and kings. Prior to the deal, each of two players places and *ante* on the table as an entrance fee or sorts. Players are then dealt one card face down (so that players only see their own cards) and asked whether they want to "bet" or "pass". If only one player bets, she wins the "pot" (both antes and the bet), if nobody bets or if they both bet then the players turn over their cards and the high card wins the pot, with an ace beating a king. If both players have the same card, they split the pot.

(a) Suppose the ante is \$2 and the bet is \$1, draw a normal form for the game (the strategies are bet or pass).

(b) What is the equilibrium?

3) Lets make a deal is the simplest possible variable sum game. Two players are faced with a lucrative deal which generates \$x if they agree to the terms of the deal. The two players will split \$x if they both agree but if one or neither can agree they will get nothing.

(a) Write down the normal form of lets make a deal, the strategies are simply saying "yes" or "no". What are the equilibria?

4) Video system coordination is a variable sum game in which firms do better by coordinating on a sort of industry standard. Here is a normal form?

	Beta	VHS
Beta	1, 1	0, 0
VHS	0, 0	1, 1

(a) Identify any Nash equilbria.

(b) Suppose both firms earn 2 instead of 1 when they both pick "VHS", how does this affect any equilibria?

5) Suppose that the strategy set for each player is the unit interval [0,1]. Payoff functions are as follows:

$$U_1(x_1, x_2)=x_1+x_2=U_2(x_1, x_2)$$

Show that this game is strictly dominance solvable, Find its strict dominance solution and its Nash equilibrium.

6) *Rock, Paper, Scissors* is a symmetric two-player, zero-sum game. The rules are simple Rock beats (i.e. smashes) Scissors, Scissors beats (i.e. cuts) Paper, and Paper beats (i.e. covers) Rock.

(a) Write down a normal form payoff matrix for this game with 1 indicating a win, 0 a tie, and –1 a loss.

(b) Identify what a *maximin/minimax* player would do.
(c) Identify any pure strategy Nash equilibria.
(d) How would you play this game?

7) Country A and Country I are at war. The two countries are separated by a series of rivers, illustrated in the figure below.

```
A ~~~ b ~~~ c
|     |     |
d ~~~ e ~~~ f
|     |     |
g ~~~ h ~~~ I
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Country I sends its navy with just enough supplies to reach A. The fleet must stop for the night at intersections (e.g. if the fleet takes the path IhebA, it must stop for the first night at h, the second night at e and the third at b). if unhindered, on the fourth day the fleet will reach A and destroy country A. Country A can send its own navy to prevent this. Country A's fleet has enough supplies to visit three contiguous intersections, starting from A (e.g. Abcf). If A's fleet catches I's fleet (i.e. if both navies stop for the night at the same place), it destroys the fleet and wins the war.

(a) List the strategies for the two countries (say A can go backwards and wants to be at d or b on the last night – otherwise the matrix is huge) and make a normal form payoff matrix (assume the winner gets 1 and the loser –1).
(b) Eliminate strictly dominated strategies. What happens to the matrix in (a)?
(c) Eliminate **weakly** dominated strategies. Note that the order matters when you eliminate weakly dominated strategies. Write down the two possible resulting matrices.
(d) Identify any Nash equilibria.

------------------ EC280 PROBLEM SET 3 --

Please answer all the following questions and try to write legibly.

1) Kids sometimes play a game called "matching pennies". The way it works is that each player turn a penny either heads up or tails up in her hand. Once both have chosen they simultaneously show their pennies. The "matcher" wins both pennies if they are both heads or both tails and the "differencer" wins if the pennies differ. Clearly this is a zero-sum game.
   (a) Write down a normal form of matching pennies and solve for any equilibrium. How should you play this game?
   (b) Suppose the game is asymmetric in that player 1 wins +2 if they match on heads and player 2 loses -2. Otherwise the game is the same. Find the new mixed strategy equilibrium.

2) In Liar's Poker an ace ranks higher than a king. There are two players. Player one is dealt a card that only she sees. She has to announce either "ace" or "king". Player 2 can either "call" one's potential bluff (saying ace when she really only has a king) or "fold". Given the likelihood of the two cards, the normal form looks like this:

	Call	Fold
Ace	0, 0	0.5, -0.5
King	0.5, -0.5	0.25, -0.25

   (a) Find any Nash equilibrium
   (b) Suppose that payoffs change to the following, what is the new equilibrium?

	Call	Fold
Ace	0, 0	0.6, -0.6
King	0.5, -0.5	0.3, -0.3

3) In a variant of Video System Coordination (from problem set 2), suppose that it matters whether players coordinate on Beta or VHS in the sense that coordinating on Beta pays 2, while coordinating on VHS pays 1. Find all three Nash equilibrium for this game and then "solve" using the notions of payoff dominance and symmetry.

4) Suppose there is a market niche game in which two firms are deciding whether to enter a new market or not. The payoffs look like this:

	Enter	Stay Out
Enter	-50, -50	100, 0
Stay Out	0, 100	0, 0

   (a) What are all the equilibria?
   (b) Suppose the game is asymmetric in that the market niche is worth only 90 to firm 2. What are the equilibria now?

5) Here is a game that arises in resource economics. There are two fishing spots, one "hot" and one "cold". The hot spot has 20 fish, whereas the cold spot has only 12. A fisher can go to either spot to fish but they can only go to one spot. If the two fishers go to the same spot they share the fish. What are the equilibria?

6) Correlated Equilibrium: Consider the Up-Down/Left-Right game played by Alphonse and Gaston, with normal form shown below.

	G Left	G Right
A Up	5,1	0,0
A Down	4,4	1,5

(a) Find all the equilibria. And the expected payoffs for both players at each equilibrium.
(b) Show that the two can achieve average payoffs of (3,3) as a Nash equilibrium if they are given a fair die.

7) Hawk-Dove-Bourgeois: Consider the following game description. There are two types in the population, those called Hawks because they are very aggressive, and those called Doves because they avoid confrontation. Hawks and Doves are mixed in the population and randomly interact in pairs. They interact to either share of fight over a prize of value, V. When two Hawks meet, they fight over the prize and each has an equal chance of winning. The cost of fighting *and losing* is C ($V<C$). When two Doves meet, they share the prize equally without fighting. When a Hawk meets a Dove, the Dove gets scared and runs away. In this case the Hawk gets the whole prize and the Dove gets nothing.

(a) Write down the normal form of this game.
(b) Find all the Nash equilibria.
(c) Now consider the addition of a third strategy, Retaliator. Retaliators behave like a Dove against another Retaliator, but, if its opponent escalates, Retaliators escalate too and act like Hawks. Last, Retaliators can bully Doves and therefore do a little better when meeting them. Assume that Retaliators get ¾ of the value in an interaction with a dove, but the two do not fight. Write down the new normal form.
(d) Find all the equilibria of the new normal form (set $V=1$ and $C=2$).
(e) Using Symmetry, Efficiency and Payoff dominance to see which equilibria seem more likely.

8) Modeling Team Production: Suppose there are $n+1$ workers who work as a team to produce output that they share equally. Workers have t_i (i=1…$n+1$) hours to spend either working or relaxing and each individual, i, chooses how many hours to work. Call this choice x_i. Workers value leisure at a constant rate of α utils and working provides output which workers value at q. For reasons that will become obvious below, assume $q/(n+1)<\alpha<q$.

(a) What is each worker's objective function? Label it π_i (hints: each worker receives a $1/(n+1)$ share of the total output from the team so what is team output? The remaining hours are worth α each.)
(b) How many hours does each worker work if they make identical choices? (hint: this should solve the problem max π_i by choosing x_i.)
(c) Show that this is a social dilemma. (hint: show that if they all work fully they all do better)

9) Modeling Team Production with Reciprocal Agents (this is hard): Now let's redo our analysis of team production making it both more and less complicated. To complicate things let's say that there is some likelihood that workers punish shirkers (i.e. those who don't work hard). We also complicate things by making effort costly. Working costs the worker b. However, to simplify things say working or shirking is now a binary choice and all workers face the exact same incentives so we can loose the index, i.. That is, we still have $n+1$ workers but now let $e \in \{0,1\}$ be the discrete choice of each worker to work or shirk. Each workers objective function can now be formalized as,

$$\frac{(1-\sigma)(1+n)q}{(1+n)} - be$$

where $(1-\sigma)$ is the probability that any team member works. The condition $(1-\sigma)q<b<q$ assures this remains a social dilemma. Now consider the case of the $1+n^{\text{th}}$ team member.

(a) When this member decides to shirk, she imposes a cost of q on the team. Therefore, what is the *net benefit of shirking*? (hint: this is the benefit of shirking minus the benefit to working)
(b) Now assume workers reciprocate the cost imposed by shirkers on the group by punishing them. Let m be the probability that any member of the team monitors. Further let s be the punishment that monitors can impose on shirkers. If there are n team members,

this means the expected punishment the $1+n^{\text{th}}$ team member can expect is *nms.* Including the expected punishment, what is the new net benefit of shirking?

(c) What is the equilibrium frequency of monitoring, m^*?

(d) Define, p as the "psychic" payoff people get from catching and punishing shirkers. Define the net benefit to monitoring as the benefit from monitoring another person minus the benefit from not monitoring. If $1-\sigma$ is the frequency of workers, then σ is the frequency of shirkers in the group. Say monitors get a benefit of p per shirker caught, but have to pay a cost, c, to monitor someone else. Write down the net benefit to monitoring.

(e) What is the equilibrium probability of anyone shirking, σ^*?

(f) Now say p is a function of the size of the group or $p(n)$ where $p'(n)<0$ so people get less benefit from punishing in larger groups (maybe because the groups are more anonymous). Formally prove that large groups will suffer from more shirking than small groups?

------------------ EC280 PROBLEM SET 4 --

Please answer all the following questions and try to write legibly.

1) Here is another version of Battle of the Networks from problem set 2 in which the actual market shares are given instead of the differences in market shares:

	Sitcom	Sports
Sitcom	55%, 45%	52%, 48%
Sports	50%, 50%	45%, 55%

Suppose now that there are three networks in Battle of the Networks. The market shares of the existing two networks have fallen 10 percent each (in each cell of the matrix) to make room for the new network. Redraw the normal form and find the solution.

2) Find the mixed strategy equilibrium of Video System Coordination (from problem set 2) with three firms.

3) In 3-player Let's Make a Deal (problem set 2) there is $40 million on the table. Player 1 gets paid twice as much as either player 2 or player 3 in the event there is a deal. Find all the equilibria you can.

4) The Nash Demand Game is a simple bargaining game in which player make demand over shares of a common "pie". If the sum of the demands can be accommodated by the size of the pie, they are. If the sum of the demands exceeds the size of the pie then nobody gets anything. Consider a pie of size 3 and a strategy space in which players can demand 0, 1, 2, or 3.
   (a) Write down the normal form and identify all the pure strategy equilibrium.
   (b) Find the symmetric pure strategy Nash equilibrium with the highest efficiency.
   (c) Can you find a mixed strategy equilibrium?

5) In Tragedy of the Commons, the commons production function is $F(X)=1.6X-0.2X^2$. The rate of return outside the commons remains 0.1. Find the symmetric Nash equilibrium when there are seven players.

6) Here's a three player sequential and simultaneous game. Say 3 department stores (A, B and C) consider opening new stores in one of two malls. The urban mall is smaller and can only accommodate 2 stores, but the rural mall is larger and can handle 3 stores. Each store prefers to be in a mall with one or more other stores over being alone in a mall because malls with multiple stores attract more customers and profits will be higher. Further, stores prefer the urban to the rural mall because urban customers have more money to spend. Each store must choose between trying to get space in the Urban mall and in the Rural mall. Say the stores rank the five possible outcomes as follows: 5, (the best) Urban mall with one other store; 4, rural mall with one or two other stores; 3, alone in urban mall; 2, alone in rural mall; and 1, (worst) alone in rural mall after getting rejected in the urban mall.

   (a1) Sequential move game - let's first assume the stores move in the following order: A then B then C. Draw the extensive form game and fill in the payoffs (A,B,C) at the terminal nodes (assume A gets screwed if too many choose U).

   (a2) List the strategies for the three stores based on the extensive form in (a1). There should be 16 strategies for C, 4 for B, and 2 for A.

   (a3) Convert the extensive form from (a1) into a normal form game. I suggest creating a matrix for B and C and then allow A to choose the matrix.

   (a4) Find any pure strategy Nash equilibria. in the two versions of the game.

   (b1) Simultaneous move game – now let's say the game is played simultaneously so that A, B, and C don't know what each other have done when making a decision. Draw a normal form for this game. It should be different from what you got in (a3).

(b2) Find any pure or mixed strategy equilibria in this game.

------------------ EC280 PROBLEM SET 5 --

Please answer all the following questions and try to write legibly. There are NO book problems this time.

1) Market demand is given by p=140-q. There are two firms, each with unit costs = \$20. Firms can choose any quantity. Find the Cournot equilibrium and compare it to the monopoly outcome and the perfect competition outcome. Why aren't the latter equilibria of the market game?
2) Suppose that in problem 1, firm 1's unit costs fall to \$10, while firm 2's don't change. This gives firm 1 a cost advantage. How much does firm 1 produce at the Cournot equilibrium. Is this more than firm 2? Why?
3) Suppose that market demand is nonlinear, taking the form $p=100-q^2$. Suppose that firms' unit costs are \$5. Find the Cournot equilibrium. What does this suggest about the properties of Cournot competition in general?
4) Prove the Cournot Limit Theorem for the following markets: (a) market demand is p=80-4q, and each firm has unit cost = \$10; (b) the market in problem 3.
5) Suppose that two firms both have average variable costs = \$50. Market demand is q=100-p. Find the Bertrand equilibrium. Would your answer change if there were three firms?
6) Same demand as in problem 5. Find the Bertrand equilibrium if firm 1 has average variable cost = \$40 and firm 2 has average variable cost = \$60. Show that the Bertrand equilibrium changes when firm 2 has average variable cost = \$90. Explain.
7) Same demand as in problem 5, only now there are two firms with average variable cost = \$50 and two firms with average variable cost = \$60. Find the Bertrand equilibrium. Is it the same as the perfectly competitive equilibrium?
8) Here is a model of cigarette product differentiation. There are n brands of cigarettes. Each brand of cigarettes, i, has the demand curve:

$$x_i = 15000 - 1000p_i - (1000n)(p_i - average\ price)$$

where p_i is brand *i's* price in dollars. Average cost is \$1/pack throughout the industry. What happens to industry profits at Bertrand equilibrium when the number of firms rises from $n=2$ to $n=3$? How does this relate to the price of Marlboros?

9) Suppose there are two countries, labeled 1 and 2. let x_i be the tariff level of country i (in percent), for i=1,2. If country i picks x_i and the other country (j) picks x_j, then country i gets a payoff of $2000 + 60x_i + x_ix_j - x_i^2 - 90x_j$ (measured in billions of dollars). Assume that x_1 and x_2 must be between 0 and 100 and that the countries set tariff levels simultaneously.

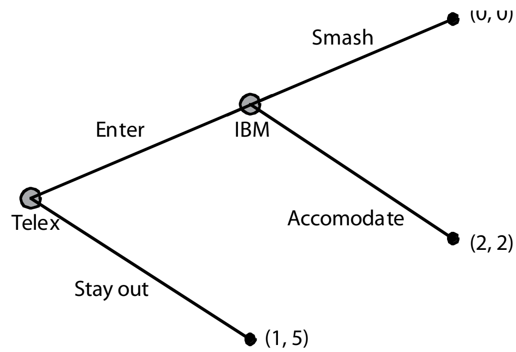
   (a) Find the best response functions for the two countries.
   (b) Compute the Nash equilibrium.
   (c) Show that the countries would be better off it they made a binding agreement to set lower tariffs (than in equilibrium).
   (d) Graph the best response functions to show the equilibrium and the social optimum.

10) Redo number 1 finding the von Stackelberg or Leader-follower equilibrium. Say firm one is the leader.

------------------ EC280 PROBLEM SET 6 --

Please answer all the following questions and try to write legibly.

1) In the market entry game, Telex versus IBM, there is an incumbent (IBM) who is asked to share the market with a newcomer (Telex). Telex goes first and decides whether to "enter" or "stay out" then IBM goes and decided to "accommodate" the entrant or "smash" her. Here is an extensive form (Note the smash payoffs are (0, 0)):



   (a) Write down a normal form and identify any pure strategy Nash equilibria.
   (b) Which equilibrium is subgame perfect (and why)?
   (c) Suppose now that IBM goes before Telex. Write down the new extensive form. What is the subgame perfect equilibrium now?

2) Consider the game of Centipede with three moves. Player 1 moves first and can "grab" all the current money \$1 or "wait". If player 1 waits, player 2 can then grab all the money \$4 or wait. On round three the money quadruples again and player one decides between grabbing or "splitting" the money with player 2.
   (a) Write down the extensive form and solve. What is the subgame perfect equilibrium?
   (b) How might play differ if you, were player 1 and you thought that player 2 was "nice"?

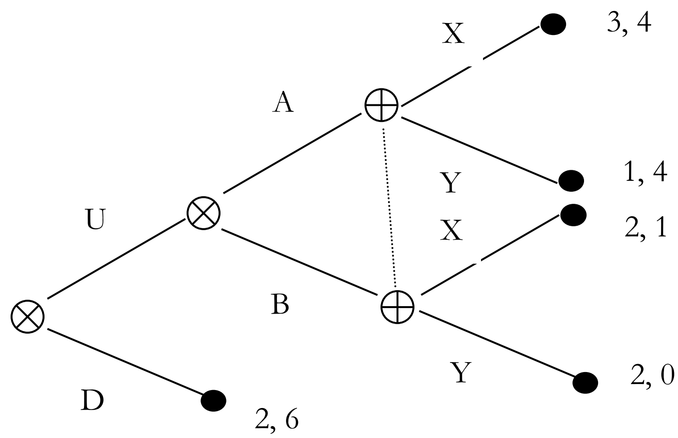
3) In the Conscription Game The army needs one of two people to join up. Player 1 has to move first and can either "volunteer" for army service or "wait" for the draft (in which case he may or may not have to serve). If he volunteers the game is over and player 1 receives a payment of b-c where b is a service bonus and c is the cost of serving (and player 2 gets 0). If player 1 waits then player 2 faces the same choice of volunteering or waiting for the draft. If player two volunteers he gets b-c and player 1 gets 0. If both players choose to wait, no sign-up bonuses are paid and the recruiter executes the draft by flipping a coin to see who has to serve (hence payoffs of both waiting are expected values of the draft).
   (a) Write down the extensive form where b=300 and c=400. What is the subgame perfect equilibrium?
   (b) Now suppose the bonus is 500 and the cost of serving is 400. What is the subgame perfect equilibrium now?

4) In cournot-Stackelberg market competition, each firm faces the market demand P=90-Q. Each firm has unit cost \$30 for each unit it ships to market. Firm 1 moves first. Find the equilibrium. Show, based on firm profits , that there is a first-mover advantage.

5) Starting from the previous problem, suppose that firm 2, which moves second, has unit cost = c. What value must c be in order for firm 2 to have the same market share as Firm 1 in the Stackelberg equilibrium? This cost advantage is a measure of how big the first mover advantage is.

6) Consider the following game in extensive form. The players are called Cross and Plus. There is an information set at Plus' move.

(a) Write down the normal form of this game. Note: Cross' strategies have two components and Plus' strategies have only one component.
(b) Find the pure strategy equilibria in the normal form.
(c) What is the subgame perfect equilibrium?



------------------ EC280 PROBLEM SET 7 --

Please answer all the following questions and try to write legibly.

1) Show that the 2-person Cournot market game in which p=130-Q, AC=10, and $Q=q_1+q_2$ played twice has a unique subgame perfect (SGP) equilibrium. Also show that the 2-person game below played three times, has a unique SGP equilibrium.

	C	D
C	3,3	0,4
D	4,0	2,2

2) Find the average payoffs of the most socially efficient subgame perfect equilibria for the following game played twice.

	E	S
E	-50,-50	100,0
S	0, 100	0,0

3) Find three strategies that yield an average payoff of (2.5, 2.5) in the following game played twice.

	A	B
A	3,3	1,4
B	4,1	0,0

4) Find a SGP equilibrium that supports monopoly payoffs for the Cournot market game played an infinite number of times where p=130-Q, $Q=q_1+q_2$, and AC=30. Say both firms have discount factors of 0.90.

5) Suppose in problem (4) that firm 1 has AC=10 and firm 2 has AC=30. Find the one-shot Cournot equilibrium. What are the profit possibilities of this game repeated infinitely often (i.e., what does a two-by-two table of monopoly versus cournot look like?).

6) Consider the following game:

		Player 2		
		X	Y	Z
	X	3, 3	0, 0	0, 0
Player 1	Y	0, 0	5, 5	9, 0
	Z	0, 0	0, 9	8, 8

(a) What are the equilibria of the stage game?
(b) If the players could write a contract to play a strategy that could be costlessly enforced by a court, what would they agree to do?
(c) Suppose, however, they can't write a contract. If the game is played for 2 periods, show there is a subgame perfect equilibrium in which {Z, Z} is played in the first period.

7) Consider the following "war of attrition" game. Interaction between players 1 and 2 takes place over discrete periods of time, starting in period 1. In each period, players choose between "stop" (S) and "continue" (C) and they receive payoffs given by the following stage game matrix:

		Player 2	
		S	C
Player 1	S	x, x	0, 10
	C	10, 0	-1, -1

However, the length of the game depends on the players' behavior. Specifically, if one or both players selects S in a period, then the game ends at the end of this period. Otherwise, the game continues into the next period. Suppose the players discount future payoffs according to the discount factor $0<d<1$. Assume $x<10$.

(a) Assume players use mixed strategies. Show that, if $d>(11-x)/(12-x)$ the infinitely repeated version of this game has an equilibrium in which players use the mixed strategy forever. But if the inequality goes the other way, the players end the war of attrition today.

------------------ EC280 PROBLEM SET 8 --

Please answer all the following questions and try to write legibly.

Evolutionary Game Theory

1) Find an ESS and draw a phase diagram for the Prisoner's Dilemma with payoffs below.

	C	D
C	3,3	0,4
D	4,0	2,2

2) Find two ESSs and draw a phase diagram for the following coordination game. Which equilibrium has a larger basin of attraction?

	A	B
A	3,3	0,0
B	0,0	2,2

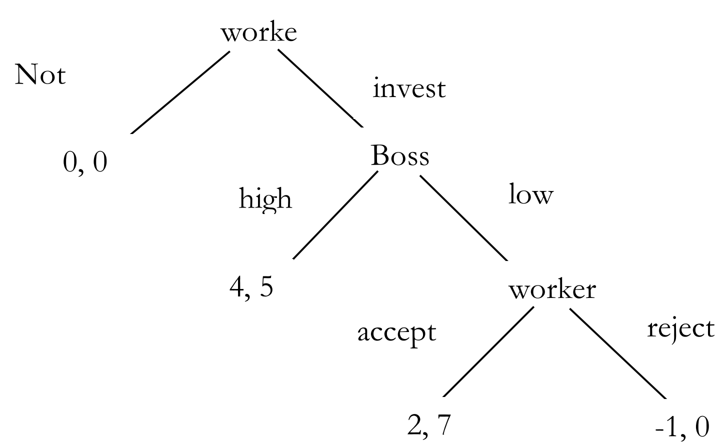
3) Find the ESSs and draw a phase diagram for the following Hawk-Dove game.

	A	B
A	-49,-49	2,0
B	0,2	1,1

4) Find any ESSs and draw a phase diagram for the following asymmetric game.

	O_2	L_2
O_1	6,4	5,5
L_1	9,1	10,0

5) Consider the following "hold-up" game expressed in extensive form:



Where workers invest in human capital or not, then bosses offer high wages or low wages and then workers accept the low wage or reject it. Note, this model implies that if the worker invests she can generate 10 units of profit in the firm, but it costs her 1 unit to invest. Further, we assume high wages are always accepted.

(a) Write down the normal form of this game when you consider the two strategies of the boss (High or Low) and only three strategies for the worker (not invest, invest accept, and invest reject).
(b) Is there a "hold-up" problem here? Why or why not.
(c) Construct a phase diagram for this game based on the replicator dynamic (note: there are two populations).
(d) Which, if any, of the equilibria are ESSs?
(e) Now assume a fraction d of the players in each population mess up and choose randomly while (1-d) of the players choose based on the replicator dynamic. What are the critical values of d such that the ESS(s) in (d) are still stable.

------------------ EC280 PROBLEM SET 9 --

Please answer all the following questions and try to write legibly.

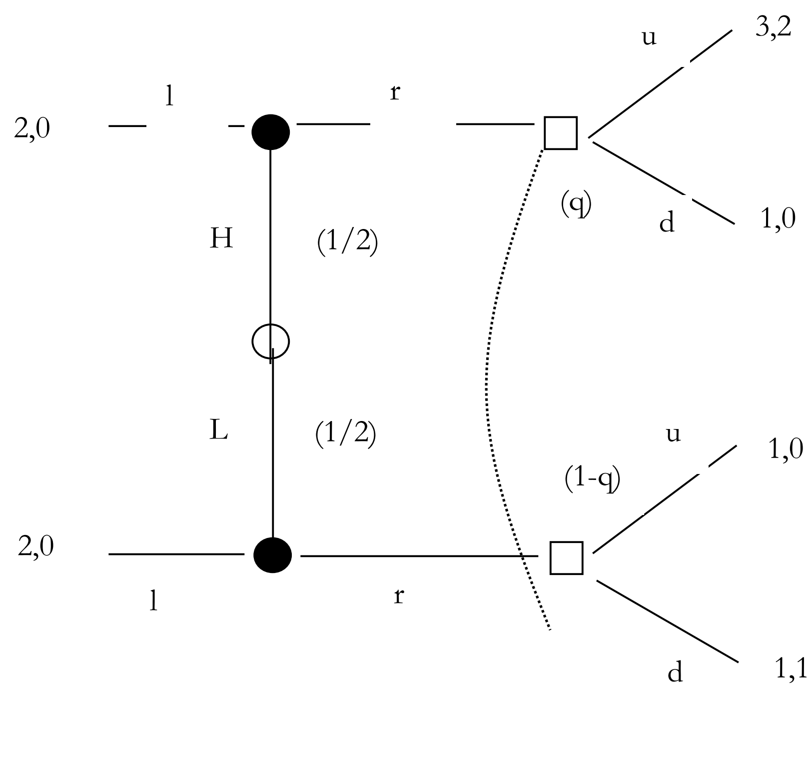
1) An art dealer has offered you a disputed Rembrandt. Art experts are evenly divided over whether this is an authentic Rembrandt (in which case it is worth $20 mil) or the work of a student of Rembrandt's (in which case it is worth $1 mil). Set this up as a signaling game and solve. The price you are being asked is $5 mil, and you are risk neutral (so you just care about expected values). It costs $100,000 to get an art critic to authenticate the painting.

2) You are considering a leveraged buyout of Corporation X. The stock of X is worth either $1/share or $5/share. The management of the company knows what it is worth, and is asking $2/share for the 10,000 outstanding shares. All you know is that the probability that the company is worth $5/share is 50 percent. Should you buy the company at this price? It costs management $50,000 to "cook the books" if it has to make the firm look better than it really is.

3) From your notes redraw the Lemons Problem with V=$5,000, W=$1,000, p=$3,000, q=$500, and a positive c of your choice. What is the sequential equilibrium of Lemons?

4) Consider the following game with asymmetric information. The open circle is Nature's initial move, closed circles are for the informed first mover and open boxes are for the uniformed second mover. Further, probabilities are listed in brackets.

   (d) Fully describe a *separating* equilibrium in this game, if one exists.
   (e) Fully describe a *pooling* equilibrium in this game, if one exists.



------------------ EC280 PROBLEM SET 10 ---

Please answer all the following questions and try to write legibly.

1) **Principal-Agent I**: Here is a straight-forward P-A problem to get you started. An agent can put in either high or low effort. If he puts in high effort the project is successful with probability 0.8, if he puts in low effort the project is successful only 0.6 of the time. Effort is costly, however. If the agent work hard it costs him 150 and if he doesn't, it only costs 100. The profit of the principal is the difference between the revenues received (600, if the project is successful and 0 otherwise) and the compensation she has to pay the agent.
    (a) Determine the optimal compensation that leads to low effort by the agent.
    (b) Determine the optimal compensation that leads to high effort.
    (c) When does the principal prefer high effort?
2) **Principal-Agent II**: Suppose an agent has the following utility function: $U = w^{1/2} - e$, where w is the wage paid and e is the effort provided. Let the agent have a reservation utility, provided by some outside option of U_{min}=3. The job is such that either the task is completed or not – it is all or nothing. But, if the agent tries hard (e=1) her chance of success is better than if she does not (e=0). Specifically, if she tries hard, there is a 0.9 probability that she completes the task successfully. If she doesn't try hard her chance of success is only 0.7. Successfully completing the task leads to an output of 100 units.
    (a) Imagine the agent owned the firm instead of working at it, would she work hard or not? Why?
    (b) If the principal could see how hard the agent worked and wanted to maximize his profit (in this case, output – wage), what would he offer her as a wage? What would her utility be?
    (c) Suppose that $U = w - e$, instead and wages are determined by the output produced such that w_{low} is paid when output is 0 and w_{high} is paid when the task is completed (and output is 100). At what level must the principal set the two wages?
3) **An Easy Second Price Auction**: Suppose that there are two bidders in a second price auction, that bidder one's value is v_1=\$5, that bidder two's value is v_2=\$4, and that all bids have to be in multiples of \$2. Draw the normal form and find any Nash equilibria.
4) **A Harder Second Price Auction**: Assume that each of "n" bidders has a private value of v_i i=1...n for the good being auctioned, that each bidder bids b_i, and that the winner only has to pay b_j – the second highest bid. Prove that bidding one's value is weakly dominant.
5) **A First Price Auction**: Assume that there are 10 people participating in a first price sealed bid auction and that it is common knowledge that private values are distributed independently and uniformly on the [0,10] interval. Derive the optimal bid, b*, of a person whose private value is \$5.
6) **A Common Value Auction**: Imagine bidding in a second price auction in which there are just two bidders and the two have a common value for the object for sale. The problem is that the bidders do not have complete information about the value of the object, they only receive signals, s_1 and s_2. For simplicity suppose these signals are drawn from a uniform distribution between \$0 and \$10.
    (a) Assume that the true value of the item is just the average of the two signals, $v=(s_1 + s_2)/2$. What is bidder 1's optimal bid? (Hint: this is harder that it first appears.

To start solve for the naïve optimal bid based on the expected value of the prize. Once you have this realize how the winner's curse is inevitable and then think about the expected size of the prize conditional on having won. That is, what can you assume about the size of the prize if you are the winner?)

7) **An All-pay auction**: Consider a simplified situation similar to question 5: There are 2 bidders in a first price auction and bidder values are independent draws from the uniform distribution over the [0,1] interval. In this case, however, the auction is also "all-pay," meaning that bidders pay their values whether they win or lose.
   (a) Can you find the optimal bid, b*? (Hint: if you set up the maximand correctly things might start cancelling. What does this tell you about the solution to the all-pay auction?)
   (b) Now think about an even simpler version of the all-pay auction in which both bidders value the item at 1. If we assume that bidding zero is the same as opting out and getting a payoff of zero, can you prove that there is no pure strategy equilibrium? (Hint: focus your attention on the symmetric case)
8) **A Raffle problem**: Though you may never have participated in an all-pay auction, you surely have participated in its close cousin, the raffle. Notice the raffle is also all-pay – you pay your expenditures on raffle tickets regardless of winning or losing. Suppose N participants each have an endowment of $10 and will consider spending $\$x$ on raffle tickets. The raffle has a fixed prize equal to $P and one's chance of winning the prize is just $x/\Sigma x$.
   (a) Write down participant i's payoff function.
   (b) Find the expenditure, x^*, that maximizes the participant's payoff (i.e., how much should participants spend on raffle tickets). (Hint: solve for the symmetric equilibrium.)
   (c) Think of the comparative statics – does it make sense how the participant will react to an increase in the number of participants and the size of the prize?